The finite-dimensional decomposition property (FDDP) for non-Archimedean Banach spaces

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Abstract

A real or complex separable Banach space X has the *finite-dimensional decomposition property* (FDDP) if there exists a sequence $(D_n)_n$ of finite-dimensional subspaces of X such that every $x \in X$ can be uniquely written as $x = \sum_{n=1}^{\infty} x_n$ with $x_n \in D_n$ for all $n \in \mathbb{N}$. Clearly, every Banach space X with a Schauder base has the FDDP; but the converse is false. Also, a closed subspace of a Banach space with the FDDP needs not have the FDDP.In the non-Archimedean context the situation differs substantially, every non-Archimedean Banach space of countable type has a Schauder base, thus, all such spaces and their closed subspaces have the FDDP.

Having in mind the special and important role played by orthogonality in the non-Archimedean theory of Banach spaces, a natural modification of the above classical concept reads as follows. Let E be a non-Archimedean Banach space of countable type. We say that E has the *orthogonal finite-dimensional decomposition property* (OFDDP) if it is the orthogonal direct sum of a sequence of finite-dimensional subspaces. We preesent several properties of the orthogonal finite-dimensional decomposition property.